TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 446

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Washington
February, 1933

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The purpose of this note is to present a method of estimating the variation of thrust horsepower with air speed, when the power unit consists of a conventional, thin, metal, fixed-pitch propeller driven by an internal-combustion engine. Such a curve is needed for the estimation of airplane performance in conjunction with a curve of horsepower required for maintaining level flight. The method is intended to be used in conjunction with data on families of propellers tested with airplane and engine, given in the form of selection charts of V/nD and propulsive efficiency plotted against C_s, the speed power coefficient. Complete familiarity with the use of such charts (reference 1) is presupposed, as well as an understanding of the adaptation of them to the determination of approximate characteristics of other propellers (reference 2, Ch. XVII); so this note may be considered an addition to those references. As far as possible standard symbols are used and will not be defined.

The nature of the problem is this. With a definite propeller whose characteristics are known or can be estimated, and which has been selected to fit an airplane having a certain engine and a certain maximum speed, the thrust horsepower is known for one air speed. With a change in air speed due to a modification of the attitude of the airplane without a change in the throttle setting there will usually be a change in revolution speed. The brake horsepower of the engine at constant throttle is a direct function of the revolution speed. Since the propulsive efficiency depends only on the ratio of the air speed to the revolution speed, both the brake horsepower and the propulsive efficiency can be found if the variation of the revolution speed with the air speed is known. The thrust horsepower is then the product of the brake horsepower and the propulsive efficiency.

The known characteristics of the propeller selected (or those of an equivalent propeller if it is necessary to estimate the propeller characteristics) consist of a curve J, that is, V/nD, against C_s, and a curve of η, the propulsive efficiency, also against C_s. To be of any value these curves must have been derived from full-scale tests in conjunction with an airplane form not too different from the one in which we are interested.

At a given altitude the brake horsepower of a conventional modern aircraft engine is almost directly proportional to the revolution speed at least for the range of revolution speeds from the rated value down to the full throttle static value. The assumption will be made that the power is directly proportional to the revolution speed at full throttle or any fixed throttle setting. The power of the engine is adversely affected by an increase in altitude; that is, by a decrease in density. The exact variation of engine power with altitude is complicated. (Reference 3.) At constant revolution speed the power or torque at any altitude may be expressed as a constant times the sea-level power. This constant, to be called f, is assumed to be a function of air conditions only and independent of the revolution speed. This assumption is justified by altitude-chamber tests, at least over the range of revolution speed likely to be experienced. The effect of a supercharger on an engine modifies its value of f and, on most air-cooled radial engines, requires a change in permissible throttle opening with altitude.

These relations are put into mathematical form ~~in~~ the following equations:

$$t.hp = b.hp \times \eta \tag{1}$$

$$b.hp_0 = K\,N \tag{2}$$

where N is revolution speed in revolutions per minute, subscript zero indicates sea level, and K is computed from the rated values of revolution speed and brake horsepower. This equation defines K.

$$b.hp = f \times b.hp_0 = f\,K\,N \tag{3}$$

$$J = V/nD$$

where V is the air speed in feet per second, n is the revolution speed in revolutions per second, and D is the propeller diameter in feet. In engineering units this equation becomes

$$J = 88\ V/ND \qquad (4)$$

or

$$V = J\ N\ D/88 \qquad (5)$$

where V is expressed in terms of miles per hour and N in terms of revolutions per minute. Equations (3) and (5) express the velocity and the brake horsepower in terms of constants and the revolution speed. The speed-power coefficient is expressed by the equation

$$C_s = V \sqrt[5]{\frac{\rho}{P\ n^2}}$$

with consistent units. With engineering units this becomes

$$C_s = \frac{0.638\ V}{b.hp^{1/5} \times N^{2/5}} \times \sigma^{1/5} \qquad (6)$$

where V is in miles per hour and σ is ρ/ρ_o.

Substituting the values of V and b.hp from equations (3) and (5)

$$C_s = \frac{0.638\ J\ D\ N}{88 \left(\frac{f\ K\ N}{\sigma}\right)^{1/5} N^{2/5}} = \frac{0.638\ J\ D}{88 \left(\frac{f\ K}{\sigma}\right)^{1/5}} \times N^{2/5}$$

$$N^{2/5} = \frac{C_s}{J} \times \frac{88}{0.638\ D} \times K^{1/5} \times \left(\frac{f}{\sigma}\right)^{1/5} \qquad (7)$$

At sea level by definition, f and σ are 1, so equation (7) may be written:

$$\text{At sea level} \quad N^{2/5} = C \times \frac{C_s}{J} \qquad C = \frac{88\ K^{1/5}}{0.638\ D} \qquad (8)$$

The value of $N^{2/5}$ may be left without further solution, as scales giving values of $N^{2/5}$ and N are to be found in references 1 and 2. The value of C is constant for a given engine at a constant throttle opening and a given propeller. The value may be calculated, but it is more conveniently found by making use of the known factors relating to the high-speed level flight condition, those being the solution of equation (8) and the values of C_s and J. The solution of equation (8) for other air speeds at sea level may be performed as follows, the prime mark being used to indicate corresponding values of the variables:

From the curves of J and η against C_s select C_s', J', and η'. From $N'^{2/5} = C \times \frac{C_s'}{J'}$, find N'. It then follows that $V' = J' N' \frac{D}{88}$, $b.hp' = K N'$ (sea level), and $t.hp' = b.hp' \eta' = K N' \eta'$.

Thus is obtained one value of V with the corresponding thrust horsepower, or horsepower available. If a slight amount of discretion is used in choosing the values of C_s' repetition of the procedure for five or six points will permit the drawing of a satisfactory curve of thrust horsepower against velocity. This curve can be drawn for any fixed throttle setting, but will usually be of interest for the maximum throttle setting only.

The accuracy of the resultant curve can, of course, be no better than that of the test data used, that of the suitability of the test data to the airplane for which it is used, or that of the plots of propeller characteristics. It is usually not possible to find N closer than 2 to 3 per cent.

At low values of C_s the curve of V/nD against C_s becomes nearly straight; it must pass through the origin. Equation (8) indicates that the revolution speed tends to a constant value at low air speed.

In order to find the variation of thrust horsepower with forward speed at other altitudes, the procedure may be repeated, reverting to equation (7) and inserting the proper values of f and σ for the altitude in question. The values of C_s and J would be found on the charts used previously. It is much simpler, however, to find the effect of altitude on the sea-level curve assuming, as will usually be the case, that such a curve has already been drawn. The method of finding this effect is a general one and is in no way limited to cases in which the sea-level curve has been found as outlined above.

Starting with equation (7),

$$N^{2/5} = \frac{C_s}{J} \times \frac{88}{0.638\ D} \times K^{1/5} \left(\frac{f}{\sigma}\right)^{1/5} \qquad (7)$$

in transferring the curve of t.hp against V to another altitude, we maintain a constant V/nD (a procedure similar to the maintenance of constant angle of attack when transferring a curve of horsepower required for level flight from sea level to some altitude). With J constant, C_s and η are fixed, and the only variables in equation (7) are N, f, and σ, or

$$N^{2/5} = \text{constant} \times \left(\frac{f}{\sigma}\right)^{1/5} \tag{9}$$

$$N = \text{constant} \times \sqrt{\frac{f}{\sigma}} \tag{10}$$

When both f and σ are 1, N is constant, which is then the sea-level revolution speed, N_o. This fact, together with the constant value of J, permits the following equations:

$$N/N_o = V/V_o = \sqrt{\frac{f}{\sigma}} \tag{11}$$

$$\text{t.hp} = f\ K\ N\ \eta \tag{12}$$

$$\text{t.hp}_o = K\ N_o\ \eta \tag{13}$$

The values of η are the same since J is constant.

$$\text{t.hp}/\text{t.hp}_o = f \times N/N_o = f\sqrt{\frac{f}{\sigma}} \tag{14}$$

Equations (11) and (14) give the factors by which the coordinates (V, t.hp) of any point on the sea-level curve must be multiplied, respectively, to obtain the curve at an altitude corresponding to the values of f and σ. With the usual unsupercharged engine, f decreases with altitude somewhat more rapidly than σ; hence the revolution speed at the same V/nD decreases slowly with altitude (the reduction is about 6 per cent at 15,000 feet).

The method tacitly assumes that the characteristics of the propeller are independent of the absolute value of the power it is absorbing and that the efficiency is a function of V/nD alone. The first assumption requires for its justification that the propeller should not twist to change effective blade angles with varying loads. Propellers of the usual thin metal type are known to twist somewhat under load so some error, not exactly determinable but surely small, will be caused in revolution speed (the error would be to underestimate), so in thrust horsepower. The second assumption requires that the propeller characteristics be independent of the angle of the shaft to the flight path, at least for the variations of angle of attack encountered with change of altitude. The independence of propeller characteristics of the angle of inclination of the shaft, at least up to 10°, has been so well established by tests in the N.A.C.A. propeller research tunnel that it is standard practice to omit tests at other than 0° inclination. This change of inclination with altitude arises from the fact that the speed at the same V/nD is less at altitude, while the true speed at the same angle of attack of the airplane increases ($V = V_0/\sigma$). The change in attitude of the airplane at constant V/nD will be of the order of 4° at 15,000 feet, becoming larger as the air speed approaches a minimum.

For the determination of curves of horsepower available from the propeller at altitude, given the sea-level curve, the method described above is simple and more accurate than others customarily used for the same purpose. The method for finding the original sea-level power available curve is only a little longer than approximate methods using generalized propeller data, and would usually more than justify the increased labor by finding the airplane performance with a propeller specifically fitting the operating conditions.

Massachusetts Institute of Technology,
Cambridge, Mass., December 1, 1932.

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